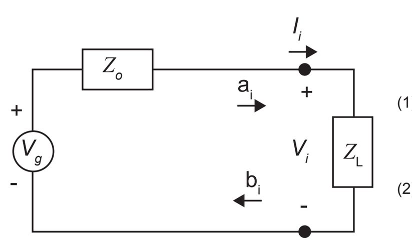
VNA Fundamentals

Circuits and Waves

The scattering parameters describe the relationship of a set of variables called a_i and b_i, the incident and reflected waves at the i^{th} port of a microwave network. These waves are defined in terms of the terminal voltage V_i and terminal current I_i with an arbitrary real impedance Z_0 (see Figure (1)).



(1) $a_i = \frac{V_i + Z_o I_i}{2\sqrt{Z_o}}$ (3) $V_i = \sqrt{Z_o}\,(a_i + b_i)$

(2) $b_i = \frac{V_i - Z_o I_i}{2\sqrt{Z_o}}$ (4) $I_i = \frac{1}{\sqrt{Z_o}}(a_i - b_i)$

Figure (1) Relation between waves and terminal voltage and current

Reflection coefficient Γ_i at port i, where the terminating impedance $Z_L = V_i/I_i$

(5) $\Gamma_i = \frac{b_i}{a_i} = \frac{V_i/I_i - Z_o}{V_i/I_i + Z_o} = \frac{Z_L - Z_o}{Z_L + Z_o}$ (6) $V_i = V_g - Z_o I_i$ (7) Return Loss (dB) at port *i*: $RL = -20Log_{10}|\Gamma i|$

Inserting equation (6) into equation (1) and squaring it yields (8) $|a_i|^2 = \frac{|Vg|^2}{4Z_o} = P_{GA}$

Where P_{GA} is available power from the voltage source Vg

The power incident minus the power reflected is given by:

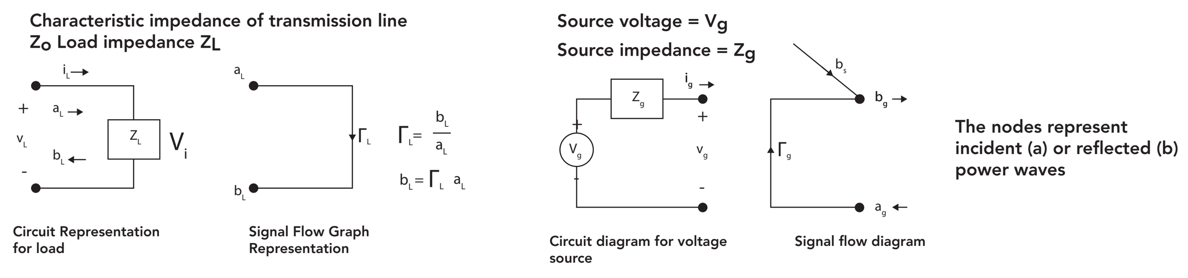
(9) $|a_i|^2 - |b_i|^2 = |a_i|^2[1 - |\Gamma_i|^2] = P_L = P_{GA} - P_R$

Where: P_L = power delivered to load
P_R = reflected power

When $Z_0 = Z_L$ then $\Gamma_L = 0$ and all of the power is transfered to the load and (10) $P_L = |a_i|^2 = P_{GA}$

S-Parameters

Signal Flow Graphics for input and output



$\Gamma_L = \frac{b_L}{a_L}$ $b_L = \Gamma_L a_L$

$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$ $i_g = \frac{V_g}{Z_g + Z_o} = \frac{a}{\sqrt{Z_o}}$ $b_g = \frac{\sqrt{Z_o}V_g}{Z_g + Z_o} + a_g\Gamma_g$ $\Gamma_g = \frac{Z_g - Z_o}{Z_g + Z_o}$

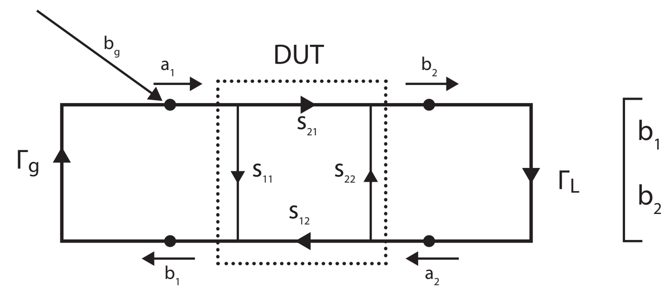
Figure (2) Input and output signal flow graph representations

The definition of the Scattering Matrix [S] is given by:

(11) **[b]=[S][a]**

In microwave circuit design, S-parameters are useful for characterization of any 2-Port network. S-parameters are determined using a reference impedance usually equal to the characteristic of the test system (generally 50 ohms).

The S-parameters are complex elements having a magnitude and phase, and are measured in terms of incident and reflected waves (a and b) using a Vector Network Analyzer (VNA).



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Figure (3) Signal Flow graphs used for 2-Port Network

The square matrix [S] represents the relationship between the vectors [a] and [b] that represents the amplitude and phase of the incident and reflected waves.

Where:

(12) $S_{11} = b_1/a_1$ forward Γ_1 at port (1) when the load reflection coefficient ($\Gamma_L = 0$) and $a_2 = 0$

(13) $S_{21} = b_2/a_1$ transmission coefficient from port 1 to port 2 when ($\Gamma_L = 0$) $a_2 = 0$, matched load on port 2

(14) $S_{12} = b_1/a_2$ reverse transmission coefficient from port 2 to port 1 when ($\Gamma_G = 0$) $a_1 = 0$, matched source on port 1

(15) $S_{22} = b_2/a_2$ reverse Γ_2 at port (2) when the reflection coefficient ($\Gamma_g = 0$) and $a_1 = 0$

The signal flow graphs can be generally solved for the reflection coefficient $\Gamma_{ii} = b_i/a_i$ and transmission function $T_{ij} = b_j/a_i$ using Masons Rule[1].
Signal flow graphs are an excellent method to analyze microwave circuits. The S-Matrix and the source are represented by the graph shown in Figure (3)

$b_g = \frac{\sqrt{Z_o}V_g}{Z_g + Z_o}$ $b_g = \frac{V_g}{2\sqrt{Z_o}}(Z_g = Z_o)$ $\Gamma_g = \frac{Z_g - Z_o}{Z_g + Z_o}$ $\Gamma_g = (0)(Z_g = Z_o)$

1 Mason, Samuel J. (July 1956). "Feedback Theory - Further Properties of Signal Flow Graphs". *Proceedings of the IRE: 920-926*

Reflection Coefficient

The reflection coefficient Γ is graphically represented as a polar display (shown in Figure 4). For passive systems the magnitude of Γ is ≤ 1. From equation (5), we have equation (16).

(16) $\frac{Z_L}{Z_O} = \left(\frac{1+\Gamma}{1-\Gamma}\right)$

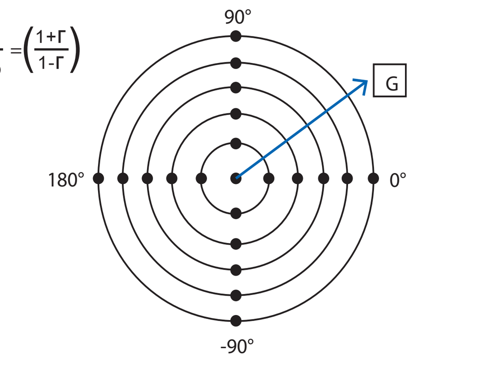


Figure (4) Polar Display of Reflection Coefficient Γ

The magnitude of the reflection coefficient $|S_{11}|$ is graphically represented as a Return Loss (equation 7) and plotted as the log magnitude versus frequency as shown in Figure (5).

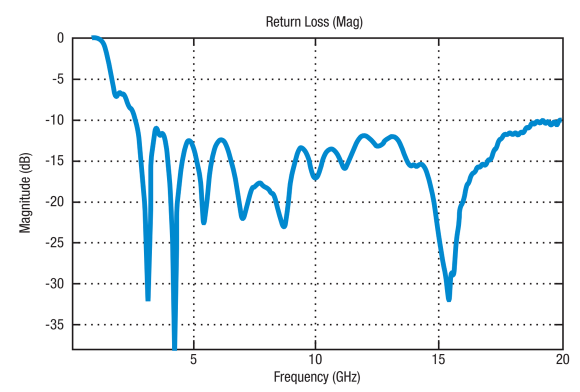


Figure (5) $|S_{11}|$ plotted as RL = 20*$\log_{10}|S_{11}|$

Smith Chart

From equation (5), we have $\frac{Z_L}{Z_O} = \left(\frac{1+\Gamma}{1-\Gamma}\right)$. We have a 1 to 1 relationship between Γ and $Z_{(\Gamma)}$.

For example: when $Z_L = Z_0$ Γ = 0, for an open or short circuit then |Γ| = 1.
The VNA maps the impedance space into the polar display of Γ as a "Smith Chart" shown in Figure 6. For every corresponding point in Γ space, there is a corresponding impedance Z_L. The VNA measures the reflection coefficient Γ and plots the impedance Z_L.
The Smith Chart is the bi-linear transformation of the reflection coefficient Γ space to the impedance Z space.

Complex Impedance Z = R+ jX

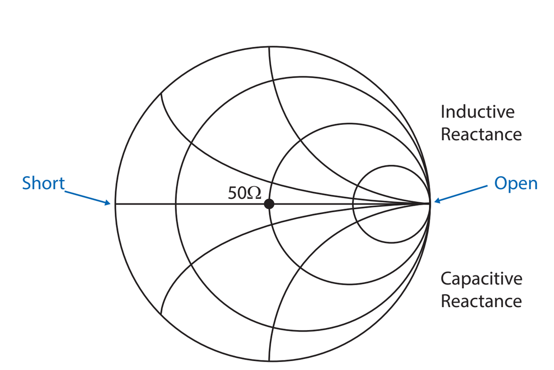


Figure (6) Smith Chart Display
Circles are constant resistance (R),
Arcs are constant reactance (X)

VNA Architecture

The dual band VectorStar VNA is comprised of a mixer based low band (70 KHz to 2.5 GHz) VNA using bridge reflectometer and a high band (>2.5 GHz) using Non Linear Transmission Line (NLTL) samplers and broad band coupler reflectometer. This design allows the VNA to operate at very low frequencies for superior time domain measurements.

For the high band (>2.5 GHz) the VNA uses four NLTL sampler receivers and associated reflectometers to measure a_1, a_2, b_1, and b_2 waves incident and reflected from the DUT. The ratios for each of the S-Parameters are calculated using data measured at each sampler. Because S-parameters are ratios, it is not necessary for the samplers to measure absolute values. For example, when measuring S_{11}, it is only necessary to know the level at b_1 relative to a_1. Figure 10 shows the block diagram for the VNA. Under normal test conditions, the input of the DUT would be attached to port 1 on the VNA, and the output of the DUT would be attached to port 2.

Notice from the block diagram that samplers a_1 and a_2 measure the power from the source via a power splitter. Sampler's b_1 and b_2 measure the response at both port 1 and port 2 via couplers at the respective ports. A normal calibration corrects for the input and output couplers, as well as any external cabling associated with a measurement setup.

Sampler a_1 measures the incident signal onto the DUT (when port 1 drives)

Sampler a_2 measures the incident signal onto the DUT (when port 2 drives)

Sampler b_1 measures the reflected signal back from the DUT (when port 1 drives)

Sampler b_2 measures the transmitted signal at the output of the DUT (when port 1 drives)

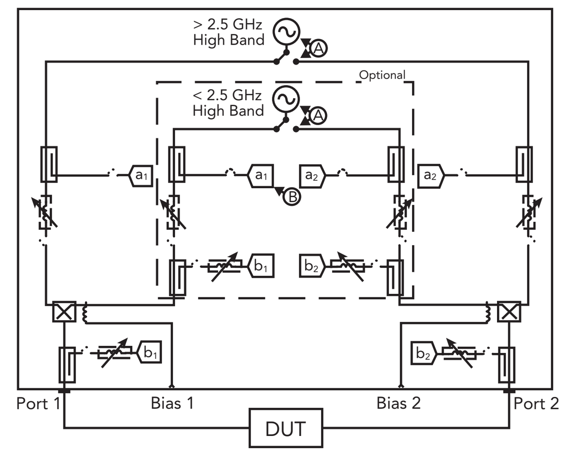


Figure (10) Block diagram for dual band VectorStar VNA

The NLTL sampler receivers have very wide inherent bandwidth (>150 GHz). The instantaneous bandwidth of the IF can be as high as 200 MHz for wide band pulse measurements when using a broad band A/D converter.

The NLTL harmonic sampler receivers offer higher dynamic range and lower conversion loss than the Step Recovery Diode (SRD) samplers and fundamental mixers whose transfer function tends to drop off by about 50 GHz as shown in the Figure (11). The NLTL technology offers higher frequency performance >150 GHz before the first null. This results in excellent dynamic range: >100 dB to 110 GHz with excellent stability.

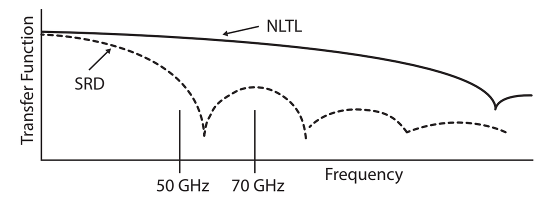


Figure (11) Comparison of the performance of NLTL sampler to harmonic mixing

Pulse Measurements

The Anritsu Vector**Star** VNA offers the option of the use of a wide band high speed (14 bit > 400 mega-sample) digitizer processor and specific pulse software to acquire and display pulsed signals.

To understand the operation of the pulse acquisition system of the Anritsu MS4640B the IF channel for each receiver is shown in Figure (12).

The VNA IF signals are generated by the non-pulse down converters in the MS4640B. When equipped with pulse mode option the standard IF system is bypassed and signals are routed to a special high-speed digitizing IF board. This board consists of analog processing (filtering, gain, calibration…) with a much wider bandwidth than the standard IF system, which enables the measurement of much narrower pulses. This board also houses the fast analog-to-digital converters, pulse generators, and digital processing components. Deep memory is used to store the data coming in from the converters. As a result, the Anritsu MS4640B can acquire long time records of more than 0.5 seconds with full resolution.

The magnitude of S_{21} for a pulsed amplifier as function of time is shown in Figure (13).

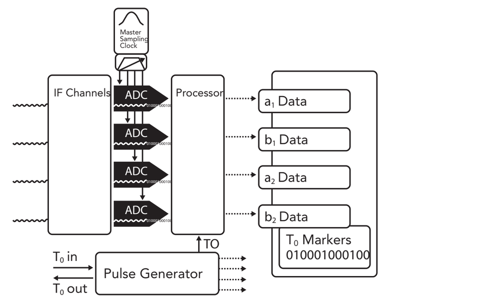


Figure (12) The data acquisition system in the MS4640B series with High Speed IF Digitizer

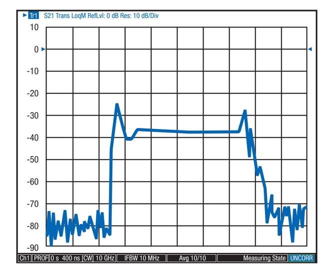


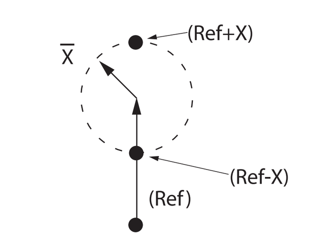
Figure (13) The pulse magnitude S_{21} response for an amplifier

Reflection Coefficient Table

SWR	Reflection Coefficient	Return Loss (dB)	dB Below Reference	Ref + X (dB)	Ref - X (dB)	Ref ±X (dB)
			Relative to Unity Reference			
17.3910	0.8913	1	1	5.5350	19.2715	24.8065
8.7242	0.7943	2	2	5.0780	13.7365	18.8145
5.8480	0.7079	3	3	4.6495	10.6907	15.3402
4.4194	0.6310	4	4	4.2489	8.6585	12.9073
3.5698	0.5623	5	5	3.8755	7.1773	11.0528
3.0095	0.5012	6	6	3.5287	6.0412	9.5699
2.6146	0.4467	7	7	3.2075	5.1405	8.3480
2.3229	0.3981	8	8	2.9108	4.4096	7.3204
2.0999	0.3548	9	9	2.6376	3.8063	6.4439
1.9250	0.3162	10	10	2.3866	3.3018	5.6884
1.7849	0.2818	11	11	2.1567	2.8756	5.0322
1.6709	0.2512	12	12	1.9465	2.5126	4.4590
1.5769	0.2239	13	13	1.7547	2.2013	3.9561
1.4985	0.1995	14	14	1.5802	1.9331	3.5133
1.4326	0.1778	15	15	1.4216	1.7007	3.1224
1.3767	0.1585	16	16	1.2778	1.4988	2.7766
1.3290	0.1413	17	17	1.1476	1.3227	2.4703
1.2880	0.1259	18	18	1.0299	1.1687	2.1986
1.2528	0.1122	19	19	0.9237	1.0337	1.9574
1.2222	0.1000	20	20	0.8279	0.9151	1.7430
1.1957	0.0891	21	21	0.7416	0.8108	1.5524
1.1726	0.0794	22	22	0.6639	0.7189	1.3828
1.1524	0.0708	23	23	0.5941	0.6378	1.2319
1.1347	0.0631	24	24	0.5314	0.5661	1.0975
1.1192	0.0562	25	25	0.4752	0.5027	0.9779
1.1055	0.0501	26	26	0.4248	0.4466	0.8714
1.0935	0.0447	27	27	0.3798	0.3969	0.7765
1.0829	0.0398	28	28	0.3391	0.3529	0.6919
1.0736	0.0355	29	29	0.3028	0.3138	0.6166
1.0653	0.0316	30	30	0.2704	0.2791	0.5495
1.0580	0.0282	31	31	0.2414	0.2483	0.4897
1.0515	0.0251	32	32	0.2155	0.2210	0.4365
1.0458	0.0224	33	33	0.1923	0.1967	0.3890
1.0407	0.0200	34	34	0.1716	0.1751	0.3467
1.0362	0.0178	35	35	0.1531	0.1558	0.3090
1.0322	0.0158	36	36	0.1366	0.1388	0.2753
1.0287	0.0141	37	37	0.1218	0.1236	0.2454
1.0255	0.0126	38	38	0.1087	0.1100	0.2187
1.0227	0.0112	39	39	0.0969	0.0980	0.1949
1.0202	0.0100	40	40	0.0864	0.0873	0.1737
1.0180	0.0089	41	41	0.0771	0.0778	0.1548
1.0160	0.0079	42	42	0.0687	0.0693	0.1380
1.0143	0.0071	43	43	0.0613	0.0617	0.1230
1.0127	0.0063	44	44	0.0546	0.0550	0.1096
1.0113	0.0056	45	45	0.0487	0.0490	0.0977
1.0101	0.0050	46	46	0.0434	0.0436	0.0871
1.0090	0.0045	47	47	0.0387	0.0389	0.0776
1.0080	0.0040	48	48	0.0345	0.0346	0.0692
1.0071	0.0035	49	49	0.0308	0.0309	0.0616
1.0063	0.0032	50	50	0.0274	0.0275	0.0549
1.0057	0.0028	51	51	0.0244	0.0245	0.0490
1.0050	0.0025	52	52	0.0218	0.0218	0.0436
1.0045	0.0022	53	53	0.0194	0.0195	0.0389
1.0040	0.0020	54	54	0.0173	0.0173	0.0347
1.0036	0.0018	55	55	0.0154	0.0155	0.0309
1.0032	0.0016	56	56	0.0138	0.0138	0.0275
1.0028	0.0014	57	57	0.0123	0.0123	0.0245
1.0025	0.0013	58	58	0.0109	0.0109	0.0219
1.0022	0.0011	59	59	0.0097	0.0098	0.0195
1.0020	0.0010	60	60	0.0087	0.0087	0.0174

- *The first three columns are conversion tables for return loss, reflection coefficient and SWR.*
- *The last four columns are values for interactions of a small phasor X with a large phasor (unity reference) expressed in dB related to reference.*

The RF Measurement Chart can be used to determine the uncertainty due to bridge/autotester VNA directivity. The "X dB Below Reference" column represents the difference between the directivity and the measured reflection (return loss). The "ref + X dB" and "ref - X dB" values are the algebraic sum of the signal as their phase relationship varies over 360°. Therefore, the peak-to-peak ripple (± X) is the total measurement uncertainty caused by the error signal.



Phasor Interaction

For example, if a 30 dB return loss is measured with a 40 dB directivity autotester, the X dB Below Reference Value is 10 dB. Ref + X dB is 2.3866 dB and ref - X dB is 3.3018 dB. The actual possible measured return loss can be between 27.6134 dB (30 - 2.3866) and 33.3018 dB (30 - 3.3018). The peak-to-peak ripple on a swept measurement will be 5.6884 dB. If the error and directivity signals are equal, ref + X dB equals 6 dB (voltage doubled causes 6 dB change) and ref - X dB becomes infinite, since the two signals are equal in amplitude and 180° outof phase (zero voltage).

Phase and Group Delay

Phase Delay

Phase delay (the argument of the S-parameters) is usually displayed in a "linear phase format" as a function of frequency as shown in Figure (7). This display shows the measurement from –180 to +180 degrees. This display method keeps the display discontinuity removed from the important 0 degree area which is used as the phase reference. The linear phase delay can be unwrapped (removal of the linear term) leaving only the deviation from linearity as shown in Figure (8).

There are several ways in which all the information can be displayed on one trace. One method is a polar display as shown in Figure (4). In this display, the radial parameter |S| is magnitude, while the rotation around the circle Φ is phase. Polar displays are used to view transmission measurements, especially on cascaded devices.

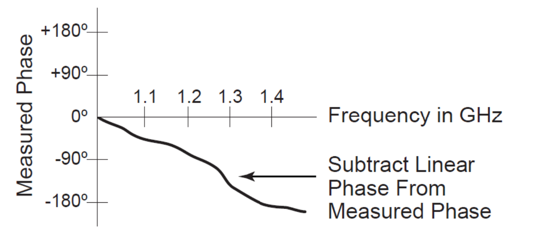


Figure (7) Phase delay with frequency of a DUT using linear format

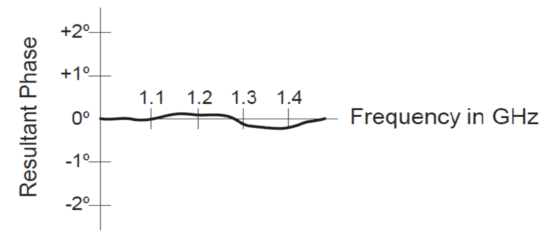


Figure (8) Residual phase variation after removal of linear term

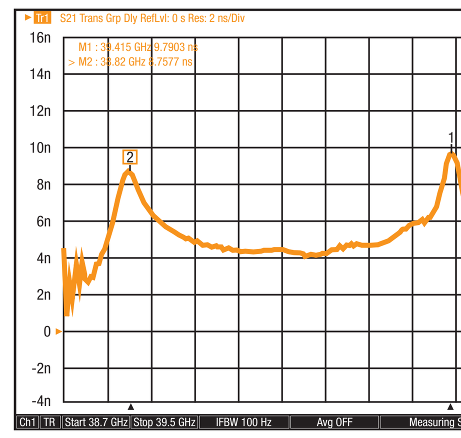


Figure (9) Group delay of a Band-Pass filter showing constant group delay in the Band-Pass and the distortion at the band edges.

Group Delay

The S_{21} parameter describes the transmission charateristics of the DUT. The argument Φ_{21} of the S_{21} represents the phase delay of the signal as it propagates through the DUT. The group delay defined by the Brillouin equation

(17) $\tau_D = -\left(\frac{\partial}{\partial\omega}\Phi_{21}\right)$ is the propagation group delay through the DUT.

The VNA uses the approximation $\tau_D = -\frac{\Delta\Phi_{21}}{\Delta\omega}$

The smaller the frequency step (Δω) the better the approximation. The plot of the group delay for a filter is shown in Figure (9).

Superposition/True Mode Stimulus

Testing Balanced Devices using Superposition or True Mode Stimulus Using a VNA

Vector network analyzers are capable of using superposition to determine the characteristics of differential passive or linear active DUT's. Each side of the differential device is stimulated in turn and the results combined mathematically. However, for non-linear differential devices, this method does not work and it is necessary to stimulate both sides simultaneously using dual sources and true mode stimulus capability. The Vector**Star** VNA does this using its internal second source and Differential**View**™ options.

Testing balanced devices using DifferentialView

- Apply true mode stimulus to differential balanced devices in a four-port mode (two ports for input and two for output)
- The differential sources are amplitude and phase adjusted to get the match-corrected signal relationship or equal amplitude and 180° phase difference at the DUT reference planes
- All balanced parameters are fully error-corrected

Measure device performance in an unbalanced state

- Set amplitude or phase to an offset relationship
- Sweep phase to determine device performance and find device anomalies

The measurements set-up is shown in Figure (14)

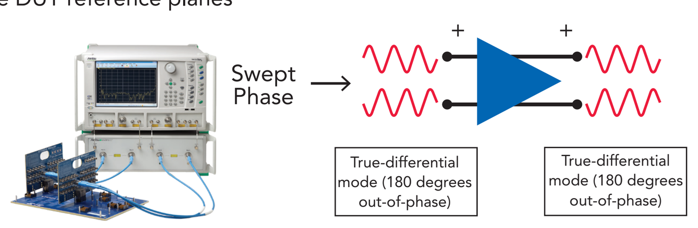


Figure (14) 4-port balanced measurement system

The stimulus input to the DUT and possible output measurements are shown below in Figure (15)

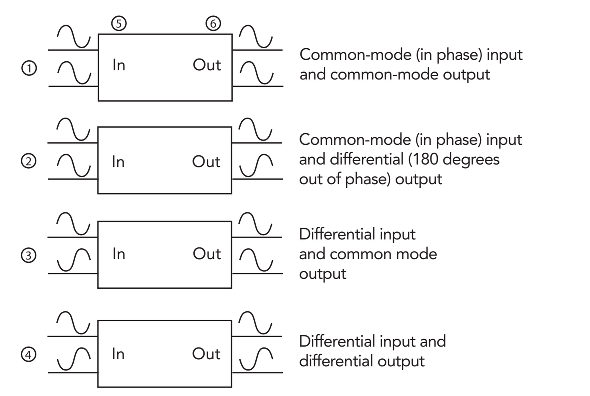


Figure (15) The new basis for analyzing mixed-mode S-parameters is shown here. With the physical ports considered as pairs, one can analyze in terms of common-mode and differential drive and common-mode and differential output.

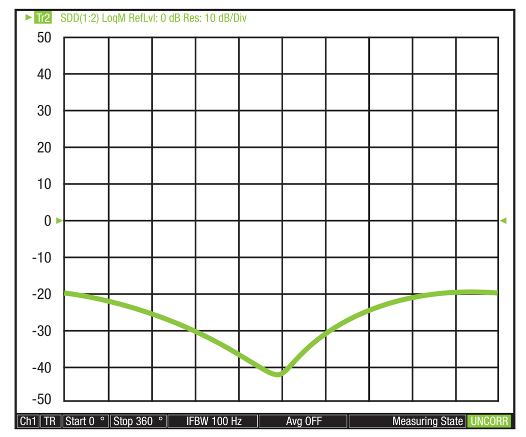


Figure (16) An example phase sweep measurement is shown here of the differential return loss (SDD) of a balun-based front-end. The match null is centered near 180 degrees as one would expect and is realtively broad.

Time Domain

Low-Pass Processing

The basic capability of the VNA is to measure in the frequency domain the signals of S-parameters of an RF or microwave device and display the result.

The Fourier transform provides a method for transforming VNA frequency domain data into the time domain mode. Low-Pass processing (harmonic frequency measurement) provides twice the spatial resolution of the Band-Pass processing. The DC term must be approximated by extra-polation and harmonic calibration is required. The shape of the real part of the step and impulse response in the Time Domain mode will show the nature of the complex discontinuity similar to that obtained by using classical Step Response measurement.

Harmonic frequency VNA data set is required:

(18) **$F_{(n)}$ = n*FL, where n =1, 2,…N and N = F_H/F_L**

Band-Pass Processing

This processing is ideal for a measurement where the DC term is not available and only discontinuity location is required. VNA data frequency set:

(19) **$F_{(n)} = F_L + n(F_H - F_L)/N$, where n = 0,1,2,3..N**

Alias Free Range

For both the Low-Pass and Band-Pass processing, the inherent alias free time range is:

(20) **τ = (N-1)/(Frequency Span)**

For example, with a 40 GHz frequency span and 1,001 data points, the Alias Free Range is:
1000/40 GHz = 25 *nanoseconds*

Time Domain Displays

Low Pass and Band Pass processing can identify the characteristics of the reflection coefficient from the real part of the S_{11}(t) display for known components.

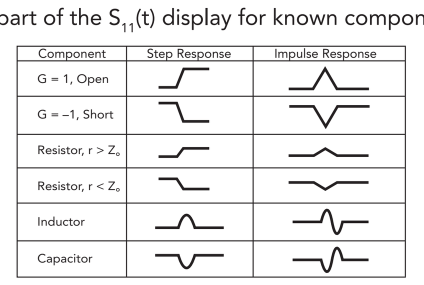


Figure (17) Example of Time Domain responses

Rise Time measurements using the VNA

These measurements require the use of Time Domain Transmission, S_{21}(t) Step response. The effective rise time for a VNA is: **$\tau_{(VNA)} = 1/(F_H - F_L)$** where F_H is the highest measurement frequency and F_L is the lowest measurement frequency. The Vector**Star** 110 GHz VNA has an effective rise time (the time between the 10% and the 90% magnitude points) of approximately 8.25 pico-seconds measured using a through line.

The rise time of a DUT is calculated using: $\tau_{DUT} = \sqrt{\tau_{MEAS}^2 - \tau_{VNA}^2}$

Non Linear Transmission Lines

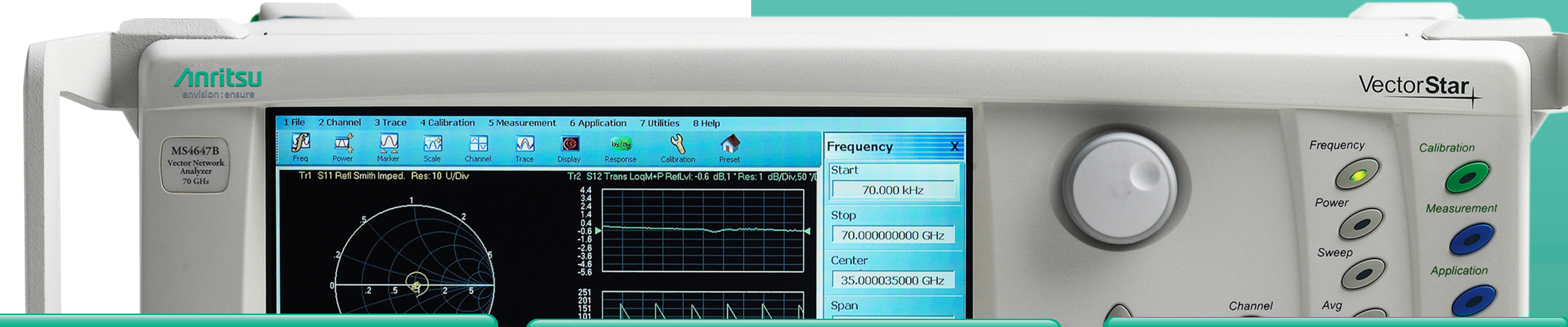
High frequency VNAs make use of harmonic samplers, or mixers, to downconvert measurement signals to intermediate frequencies (IF) before digitizing them. Such components play a critical role in VNAs, because they set the bounds on important parameters like conversion efficiency, receiver compression, isolation between measurement channels and spurious generation at the ports of a device under test.

Mixers tend to be the down converter of choice in the RF domain, due to mainly to their simpler local oscillator (LO) drive system and enhanced spur management advantages. At microwave and millimeter-wave frequencies, mixer performance tends to fall off. Today with Anritsu technology it is possible to employ Non Linear Transmission Line (NLTL) based harmonic sampling from microwave to millimeter wave frequencies.

A NLTL is comprised of a transmission line periodically loaded with varactors, where the capacitance nonlinearity arises from the variable depletion layer width, which depends both on the DC bias voltage and on the AC voltage of the propagating wave. An equivalent circuit model of one section of an NLTL is shown in the slide.

NLTLs have successfully been used to generate ultrashort electrical pulses and transients from a sinusoidal input signal. At large signal levels, waveform steepening occurs for the proper choice of input waveform and soliton generation can be achieved by balancing steepening and dispersion. Due to observable compression of the signal slope a NLTL is also often called Shockline. In the frequency domain this pulse compression results from the generation of a large number of harmonics with suitable phase relationship. The output of such Shockline is providing a very broadband harmonic rich spectrum of equidistant spectral lines. The closest line to our signal under test is used for the above mentioned harmonic sampling.

More detailed information about Anritsu NLTL and harmonic mixer application can be found in the article: "A matter of scale", April, 10th 2012, Karam Noujeim, Jon Martens and Tom Roberts



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